# ALGEBRAIC ANALYSIS OF THE MAGNIFICATION EFFECT IN THE PURE THEORY OF INTERNATIONAL TRADE

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B.A., Seoul National University, Korea, 1980

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A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF ARTS

Department of Economics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1987

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Amid the wide-spread debate[1] in the 1930s and the early 1940s over whether free trade hurt the standard of living of workers in the U.S., where labor is a scarce factor, W. F. Stolper and P. A. Samuelson put forward a conclusive thesis as to the effect of trade upon the returns to production factors. With certain assumptions, their proposition, which is known as the Stolper-Samuelson Theorem, states;

> An increase in the relative price of one commodity raises the real return of the factor used intensively in producing that commodity and lowers the real return of the other factor, regardless of which good the sellers of the factor prefer to consume.[2]

The point of this proposition concerning the changes in the relative returns to factors casts no doubt. Even before Stolper and Samuelson came out with the proposition, this income-distributional effect of the change in the terms of trade had been an agreed-upon belief among the economists in those days. The famous Heckscher-Ohlin Theorem indicated the tendency toward an equalization of factor prices between trading countries even though factors are assumed to be immobile across countries(Stolper and Samuelson). However, a question arises with respect to the definitive statement of the proposition of Stolper and Samuelson that the direction of the changes in real returns to factors does not

depend on which good is consumed intensively by such owners of different factors as laborers, landowners, etc. This statement is in fact in direct contrast to the common belief as to the relationship between the changes in relative prices of commodities and real income. If the factor owners, who might be adversely affected by trade, spend a very large share of their income on imports, their real income rises unambiguously due to the lower price of the imports. But the proposition precludes this possibility. Why does not the consumption pattern of the owners of different factors affect the change in real return to factors? Stolper and Samuelson did not answer this question clearly. It was not until Ronald Jones introduced the concept of "magnification effect" that a definite answer was given to the question. According to Jones, there is a magnification effect in the relationship between the commodity prices and the real returns to factors. In other words, if the price ratio of commodities changes, then the resulting change in the ratio of factor rewards is more magnified than the change in the price ratio of commodites that caused it. For example, if the price of a labor-intensive commodity rises, wage rates increase by a proportionally greater extent than the commodity price.

The magnification effect exists not just in the relationship between the relative changes in commodity and fac

tor prices but also in the relationship between factor endowments and levels of output. Any change in the ratio of factor endowments results in more magnified change in the ratio of levels of outputs , each of which uses different factors intensively. More specifically. if the labor endowment expands more than the capital supply, then the output of labor-intensive commodity grows even in a larger proportion than the labor endowment does.

The proposition of the magnification effect played an important role in the theory of international trade, clarifying two basic relationship in the theory between commodity and factor prices on the one hand and between factor endowments and commodity outputs on the other. It laid a firm thorectical ground not only to the Stolper-Samuelson Theorem but to another core theorem of Heckscher-Ohlin Model, the Rybczynski Theorem, that refers to the relationship between factor endowments and commodity outputs.(3) The purpose of this paper is to analyze algebraically the magnification effect that Jones dealt with roughly in his thesis. Section 2 and 3 describe the basic and the extended models for the analysis of the magnification effect. In Section 4, I discuss the magnification effect with a very simple assumption of fixed input-output coefficients. I move the discussion of the magnification effect on to a more general case of variable input-output coefficients in Section 5. In

concluding section, I indicate briefly the role of the proposition of the magnification effect in the theory of international trade.

## 2. The Basic Model

The model used for the discussion of the magnification effect is the simple general equilibrium model of production that has been extensively applied to the most analyses in the pure theory of international trade. Assume a perfectly competitive economy with two factors of production, labor(L) and capital(K), and two distinct commodities, clothing(C) and food(F), produced by the two factors in combination. Both factors are perfectly divisible, mobile and in some degree substitutible. The total available factors are fully employed in production of the two commodities. Both industries have linearly homogeneous production function, which means constant return to scale. There is no complete specialization in either clothing or food production. The factor intensities of the two industries are assumed to be different from each other such that clothing industry is labor-intensive while food industry is capital-intensive.

Under the assumption of full employment, total factor demand is equal to total factor endowments in equilibrium. Hence, the equilibrium conditions for full employment are described by two following equations.

$$L_C + L_F = L$$
$$K_C + K_F = K$$

where $L_C$ and $K_C$ denote the factors used for the production of clothing while $L_F$ and $K_F$ stand for those for food production. With the assumed production function of constant return to scale, these equations can be easily transformed as follows.

$$a_{LC}y_C + a_{LF}y_F = L \qquad (1)$$
$$a_{KC}y_C + a_{KF}y_F = K \qquad (2)$$

where $a_{LC}$ and $a_{LF}$ represent labor-per-unit-output coefficients, and $a_{KC}$ and $a_{KF}$ denote capital-per-unit-output coefficients while $y_C$ and $y_F$ refer to the levels of outputs for clothing and food respectively. These equations explain the relationship between factor endowments and commodity outputs. The two inputs are combined with outputs by the technology described in terms of input-output coefficients.(4) This pair of equations is called the full-employment conditions of the model.(5)

Under our assumption of perfect competition, profits of firms are driven to the zero level in equilibrium. In other words, total revenues in each industry are equal to total production costs for the two commodities. Hence,

$$L_Cw + K_Cr = p_Cy_C$$

$$L_F w + K_F r = p_F y_F$$

where w and r denote wage and rental rates while $p_C$ and $p_F$ represent the competitive market prices of clothing and food respectively. This pair of equations in total magnitude can reduce to those in unit cost and price under the assumption of linearly homogeneous production function as follows.

$$a_{LC} w + a_{KC} r = p_C \tag{3}$$

$$a_{LF} w + a_{KF} r = p_F \tag{4}$$

The equations, (3) and (4), which describe the equality between unit cost and price of the commodities in equilibrium under the assumption of pure competition, is called the competitive profit conditions.[6]

In the model described above that consists of the two pairs of equations, there are four parameters determined exogenously: total available endowments of the two factors(L, K), and competitive market prices of the two commodities($p_C$, $p_F$). With those parameters, the model serves to determine eight unknown choice variables: the levels of outputs of the two commodities($y_C$, $y_F$), the rate of returns to the two factors(w, r), and four factor allocation coefficients for the two industries($a_{LC}$, $a_{LF}$, $a_{KC}$, $a_{KF}$).

3. The Extended Model

The analysis of the magnification effect involves the comparative statics, where the effect of a change in parameters on endogenous variables in a model is tested, since the magnification effect involves the difference in relative changes between factor endowments and commodity outputs on the one hand, and between commodity and factor prices on the other. Therefore, for our discussion, the equations describing the basic relationships among variables should be transformed into the equations for the relationships among relative changes in variables. Let's look at the equation (1), the relationship between labor endowment and levels of outputs. Reproduce the equation:

$$a_{LC}y_C + a_{LF}y_F = L$$

Differentiate this equation totally:

$$a_{LC}dy_C + y_C da_{LC} + a_{LF}dy_F + y_F da_{LF} = dL$$

Rearrange this expression:

$$a_{LC}dy_C + a_{LF}dy_F = dL - (y_C da_{LC} + y_F da_{LF})$$

By simple algebraic manipulation, this equation can be written as[7]:

$$\frac{a_{LC}y_C}{L}.\frac{dy_C}{y_C} + \frac{a_{LF}y_F}{L}.\frac{dy_F}{y_F} = \frac{dL}{L} - \left(\frac{a_{LC}y_C}{L}.\frac{da_{LC}}{a_{LC}} + \frac{a_{LF}y_F}{L}.\frac{da_{LF}}{a_{LF}}\right)$$

If we let a $\wedge$ over a variable represent the relative change in that variable such that $\hat{y} = dy/y$, $\hat{L} = dL/L$, etc.,[8] the equation above can be written as:

$$\frac{a_{LC}y_C}{L}\hat{y}_C + \frac{a_{LF}y_F}{L}\hat{y}_F = \hat{L} - (\frac{a_{LC}y_C}{L}\hat{a}_{LC} + \frac{a_{LF}y_F}{L}\hat{a}_{LF})$$

The coefficients of the $\hat{y}$'s and the $\hat{a}$'s are the fractions of the labor force used to produce clothing and food respectively. These two fractions add to unity under the assumption of full employment.[9] Define these fractions as $u_{LC}$ and $u_{LF}$ respectively. Then the equation above is written as:

$$u_{LC}\hat{y}_C + u_{LF}\hat{y}_F = \hat{L} - (u_{LC}\hat{a}_{LC} + u_{LF}\hat{a}_{LF}) \qquad (1.1)$$

Apply the same reasoning to equation (2) for capital constraint:

$$u_{KC}\hat{y}_C + u_{KF}\hat{y}_F = \hat{K} - (u_{KC}\hat{a}_{KC} + u_{KF}\hat{a}_{KF}) \qquad (2.1)$$

Now turn to the equations for the competitive profit conditions that describe the relationship between factor and output prices. Reproduce the pair of equations here:

$$a_{LC}w + a_{KC}r = p_C \qquad (3)$$

$$a_{LF}w + a_{KF}r = p_F \qquad (4)$$

This pair of equations has a similar structre to the equations for the full-employment conditions. Hence, the same

reasoning applied to the equations, (1) and (2), can be applied to these equations in discussion. Differentiate both equations totally and use the same notation applied to the equations, (1) and (2).

Then:

$$\frac{a_{LC}w}{p_C}\hat{w} + \frac{a_{KC}r}{p_C}\hat{r} = \hat{p}_C - (\frac{a_{LC}w}{p_C}\hat{a}_{LC} + \frac{a_{KC}r}{p_C}\hat{a}_{KC})$$

$$\frac{a_{LF}w}{p_F}\hat{w} + \frac{a_{KF}r}{p_F}\hat{r} = \hat{p}_F - (\frac{a_{LF}w}{p_F}\hat{a}_{LF} + \frac{a_{KF}r}{p_F}\hat{a}_{KF})$$

The coefficients of the $\hat{w}$'s, the $\hat{r}$'s, and $\hat{a}$'s in the first equation imply the distributive shares of each factor in the clothing industry. For example, $(a_{LC}w/p_C)$ is the fraction of each dollar's worth of clothing that is paid for wages in a competitive equilibrium of zero profit. Hence, the sum of these two fractions must be unity.(10) Let $v_{LC}$ denote $(a_{LC}w/p_C)$ and $v_{KC}$ for $(a_{KC}r/p_C)$, and so on. Then the equations above are reduced to:

$$v_{LC}\hat{w} + v_{KC}\hat{r} = \hat{p}_C - (v_{LC}\hat{a}_{LC} + v_{KC}\hat{a}_{KC}) \qquad (3.1)$$
$$v_{LF}\hat{w} + v_{KF}\hat{r} = \hat{p}_F - (v_{LF}\hat{a}_{LF} + v_{KF}\hat{a}_{KF}) \qquad (4.1)$$

This pair of equations indicates the relationship in the relative changes between factor and commodity prices in a competitive equilibrium.

All the four equations, (1.1) through (4.1), are in the

same structure as the basic equations except its term for relative changes. These equations are defined as the equations of change. They are the major instrument for the analysis of the magnification effect on which all the subsequent discussions are based.

## 4. The Magnification Effect with Fixed Coefficient

In this section, I will discuss the magnification effect with a very simple assumption as to the nature of technology. Technology is assumed to employ only a fixed proportion of capital and labor in each industry although the factor intensities of the two industries are different as assumed earlier. The discussion with this simple assumption will provide the framework for the subsequent analysis with more general assumption of variable coefficients.

Under the assumption of the technology with fixed coefficients, the input-output coefficients appeared in the model are constant whatever commodity and factor prices, factor endowments, and levels of output are. Therefore, all the a's in the equation of change must be zero. Hence, the equations of change should be written accordingly:

$$u_{LC}\hat{Y}_C + u_{LF}\hat{Y}_F = \hat{L} \qquad (1.2)$$

$$u_{KC}\hat{Y}_C + u_{KF}\hat{Y}_F = \hat{K} \qquad (2.2)$$

$$v_{LC}\hat{w} + v_{KC}\hat{r} = \hat{p}_C \qquad (3.2)$$

$$v_{LF}\hat{w} + v_{KF}\hat{r} = \hat{p}_F \qquad (4.2)$$

Let us work with the first pair of equations for full-employment conditions. As dicussed earlier, the u's in the equation (1.2) are the fractions of the labor force used to produce clothing and food respectively, the sum of which must be unity under full-employment equilibrium. Similarly, the u's in the equation (2.2) are the fractions of the capital for clothing and food, which add to unity. Hence, each of these equations states that the relative changes in factor endowments are bounded by the relative changes in the outputs of the two commodities.

Subtract (2.2) from (1.2) to obtain:

$$(u_{LC} - u_{KC})\hat{Y}_C + (u_{LF} - u_{KF})\hat{Y}_F = (\hat{L} - \hat{K})$$

Since $u_{LF} = (1 - u_{LC})$ and $u_{KF} = (1 - u_{KC})$, the coefficient of $\hat{Y}_F$ is $-(u_{LC} - u_{KC})$. Therefore, by a simple algebraic manipulation,:

$$(\hat{Y}_C - \hat{Y}_F) = \frac{1}{u_{LC} - u_{KC}} (\hat{L} - \hat{K})$$

This equation can be further simplified as follows because $(u_{LC} - u_{KC})$ is equal to the determinant of the coefficient matrix of the equations, (1.2) and (2.2).(11)

$$(\hat{Y}_C - \hat{Y}_F) = \frac{1}{|U|} (\hat{L} - \hat{K}) \qquad (5.1)$$

where $|U|$ is the determinant of the coefficient matrix. As $|U|$ is a difference between the two fractions, $|U|$ is also a

fraction.

Now look at the factor intensity assumption of the model. I assumed that the clothing industry is relatively more labor-intensive. In terms of input-output coefficients, this assumption is expressed as[12]:

$$\frac{a_{LC}}{a_{KC}} > \frac{a_{LF}}{a_{KF}}$$

If we multiply both sides of this inequality by $a_{KC}$ and $a_{KF}$, then:

$$a_{LC} \cdot a_{KF} > a_{LF} \cdot a_{KC}$$

because both $a_{KC}$ and $a_{KF}$ have positive values. Therefore,

$$(a_{LC} \cdot a_{KF} - a_{LF} \cdot a_{KC}) > 0$$

Similarly, if the clothing industry is relatively more capital- intensive, then:

$$(a_{LC} \cdot a_{KF} - a_{LF} \cdot a_{KC}) < 0$$

The determinant of the coefficient matrix of the equations, (1.2) and (2.2), is expressed as:

$$|U| = (u_{LC} \cdot u_{KF} - u_{LF} \cdot u_{KC})$$

By the definition given to the u's in Section 3:

$$|U| = \frac{Y_C \cdot Y_F}{L.K} (a_{LC} \cdot a_{KF} - a_{LF} \cdot a_{KC})$$

Hence, the sign of $|U|$ determines which commodity is relatively more labor-intensive. Since the clothing industry is assumed to be relatively more labor-intensive, $|U|$ must be positive. Moreover, $|U|$ is a positive fraction because it is shown to be positive earlier. Therefore, $(1/|U|)$ in the equation (5.1) is larger than unity. It implies that any change in the labor/capital endowment ratio leads to a greater relative change in the output ratio.

Suppose that both endowments of labor and capital expand at the same rate, that is, $\hat{L} = \hat{K}$. Then the relative output change in one commodity is equal to that in the other commodity, that is $\hat{Y}_C = \hat{Y}_F$ according to the equation (5.1). In other words, in the case of a balanced growth in factor endowments, both commodity outputs expand at the identical rate.

In addition to the equation (5.1), it is shown earlier that each of $\hat{L}$ and $\hat{K}$ is a positive weighted averages of $\hat{Y}_C$ and $\hat{Y}_F$. Hence, if the labor endowment expands more than the capital supply, that is, $\hat{L} > \hat{K}$, then all the variables of the relative changes in factor endowment and commodity output can be arranged in a sequence as follows according to their sizes.

$$\hat{Y}_C > \hat{L} > \hat{K} > \hat{Y}_F \qquad (7.1)$$

In an opposite case where the capital supply expands more than the labor endowment, the whole sequence of the inequality (7.1) is reversed:

$$\hat{Y}_F > \hat{K} > \hat{L} > \hat{Y}_C$$

These two sequences of inequalities clearly show the magnification effect of factor endowments on commodity outputs. If there is any change in the ratio of labor to capital endowments, the resulting change in the ratio of commodity outputs is more magnified than the change in the factor-endowment ratio that caused it.

One of the core theorems of the Heckscher-Ohlin Model, the Rybczynski Theorem, deals with the magnification effect discussed here. The theorem states that an increase in one productive factor with constant endowment of the other results in a greater than proportionate increase in the total output of the commodity which uses that factor intensively, and actual decline in the total output of the other good if commodity prices are held constant. Returning to the inequality (7.1), let $\hat{L}$ be positive and $\hat{K}$ equal zero in order to reproduce the case that the Rybczynski Theorem describes. Then $\hat{Y}_F$ must be negative. This reasoning indicates that when only the labor endowment expands, the output level of food, which is assumed to be capital intensive, declines.

Therefore, the Rybczynski Theorem apparently refers to a special case of the magnification effect where only one factor expands with the other factor held constant.

Next, turn to the pair of equations, (3.2) and (4.2). The reasoning applied to the equations, (1.2) and (2.2), works equally well in the analysis of these equations due to the similar structure of the two pairs of equations. As indicated in Section 3, the v's in the equation (3.2) are the distributive shares of each factor in the clothing industry, the sum of which equals unity in a competitive equilibrium. In a similar fashion, the v's in equation (4.2) are the distributive shares of each factor in the food industry, which must add to unity. Accordingly, each of the two equations indicates that the relative change in the commodity prices must be a positive weighted average of the relative change in the factor prices. The relative change in the factor prices are bounded by the relative change in the commodity prices.

Subtracting (4.2) from (3.2) yields:

$$(v_{LC} - v_{LF})\hat{w} + (v_{KC} - v_{KF})\hat{r} = (\hat{P}_C - \hat{P}_F)$$

Since $v_{KC} = (1 - v_{LC})$ and $v_{KF} = (1 - v_{LF})$, the coefficient of $\hat{r}$ is equal to $-(v_{LC} - v_{LF})$. Hence, by a simple algebraic manipulation,:

$$(\hat{w} - \hat{r}) = \frac{1}{v_{LC} - v_{LF}} (\hat{p}_C - \hat{p}_F)$$

Because $(v_{LC} - v_{KC})$ is equal to the determinant of the coefficient matrix of the equations, (3.2) and (4.2), this expression can be simplified as follows.(13)

$$(\hat{w} - \hat{r}) = \frac{1}{|V|} (\hat{p}_C - \hat{p}_F) \qquad (5.2)$$

where $|V|$ is the determinant of the coefficient matrix. The determinant of the coefficient matrix can be expressed as:

$$|V| = (v_{LC} \cdot v_{KF} - v_{LF} \cdot v_{KC})$$

By the definition of the v's:

$$|V| = \frac{w \cdot r}{p_C \cdot p_F} (a_{LC} \cdot a_{KF} - a_{LF} \cdot a_{KC})$$

The sign of $|V|$, whether it has positive or negative value, depends on the sign of $(a_{LC} \cdot a_{KF} - a_{LF} \cdot a_{KC})$ because all the other terms of the right side of this equation are positive. As discussed earlier, since the clothing industry is assumed to be labor intensive,

$$(a_{LC} \cdot a_{KF} - a_{LF} \cdot a_{KC}) > 0$$

Therefore, $|V|$ has positive sign. Again, $|V|$ is a fraction because it is a difference between the two fractions, $v_{LC}$ and $v_{LF}$. For this reason, $(1/|V|)$ in the equation (5.2) is larger than unity. It means that any change in the commodi-

ty price ratio causes a greater change in the fator price ratio. Moreover, the relative change in the commodity price is equal to a positive weighted average of the relative price changes in the two factors.

Suppose that the prices of two commodity rise proportionately ($\hat{p}_C = \hat{p}_F$). Then, the prices of the two factors rise also at the same rate each other ($\hat{w} = \hat{r}$), according to the equation (5.2). The pure inflation lead to the identical increase in the prices of both factors. However, if the price of clothing, the labor-intensive commodity, rises higher than the food price ($\hat{p}_C > \hat{p}_F$), then each variable of the relative change in prices is ranked according to its size as follows.

$$\hat{w} > \hat{p}_C > \hat{p}_F > \hat{r}$$

In an opposite case where the price of food rises higher than that of colthing, the sequence of the inequality above is reversed as:

$$\hat{r} > \hat{p}_F > \hat{p}_C > \hat{w}$$

These two sequences of inequality show the magnification effect of commodity prices on factor prices. If there is any change in the price ratio of two commodities, the resulting change in the price ratio of two factors is greater than the initial change in commodity price ratio that caused

it.

It is clear that an increase in the relative price of a commodity increases the return to the factor used intensively in producing that commodity, and lowers the return to other factor as the Stolper-Samuelson Theorem states. In other words, if the relative price of the the labor-intensive commodity, clothing in our example, rises, then wage rates increase (if $\hat{p}_C > 0$, then $\hat{w} > 0$). Moreover, if the relative price of the capital-intensive good is held constant, rental rates decline (if $\hat{p}_F = 0$, then $\hat{r} < 0$). But what happens to real wage rate, for example, if laborers spend a large share of their income on the labor-intensive good, the price of which rises as questioned in Section 1? The increased income of laborers resulting from higher wage rates are obviously offset by their increased expenditure on the price-rising commodity. However, since wages rise by a proportionally greater extent than the price of the labor-intensive commodity due to the magnification effect ($\hat{w} > \hat{p}_C$), the rise in the price of that commodity leads to an unambiguous increase in real wage rate regardless of which commodity the laborers consume. Even when laborers spend all their income on the labor-intensive commodity, their real wages increase by ($\hat{w} - \hat{p}_C$) in our example. The same reasoning applies equally to the case of the price change in the capital-intensive commodity. Therefore, a rise in the

relative price of a commodity results in the increase in the real return to the factor used intensively in its production whatever the consumption pattern of the factor owners is.

## 5. Magnification Effect with Variable Coefficients

In Section 4, I dicussed the magnification effect with a very simple assumption of technology characterized by fixed input-output coefficients. Now let us move our discussion of the magnification effect on to a more general case of variable input-output coefficients. In this case, the productive techniques are capable of responding to changes in market conditions. For example, if wage rates rise, capital is substituted for labor in both industries. In other words, factor proportions in producing both commodities vary according to factor prices.

Again in this case, the equations of change, (1.1) through (4.1), are the basic instrument in discussing the magnification effect with variable input-output coefficients. For convenience, the equations of change are reproduced here.

$$u_{LC}\hat{Y}_C + u_{LF}\hat{Y}_F = \hat{L} - (u_{LC}\hat{a}_{LC} + u_{LF}\hat{a}_{LF}) \qquad (1.1)$$
$$u_{KC}\hat{Y}_C + u_{KF}\hat{Y}_F = \hat{K} - (u_{KC}\hat{a}_{KC} + u_{KF}\hat{a}_{KF}) \qquad (2.1)$$
$$v_{LC}\hat{w} + v_{KC}\hat{r} = \hat{p}_C - (v_{LC}\hat{a}_{LC} + v_{KC}\hat{a}_{KC}) \qquad (3.1)$$
$$v_{LF}\hat{w} + v_{KF}\hat{r} = \hat{p}_F - (v_{LF}\hat{a}_{LF} + v_{KF}\hat{a}_{KF}) \qquad (4.1)$$

In all the equations above, the $\hat{a}$'s are no longer zero since the input-output coefficients are now variable. Hence, unlike the fixed coefficient case, each of the equations has the additional term bracketed on its right side.

Let us consider at first the magnification effect of commodity prices on factor rewards by working with the second pair of equations, (3.1) an (4.1). Assume that technology in both industries is bounded by the law of diminishing marginal rate of technical substitution (MRTS) between the two factors. Geometrically, the production isoquants in the two industies are assumed to be convex to the origin in input space. Under this assumption of convex isoquants, the optimal factor proportion to minimize production costs occurs where the ratio of the prices of labor and capital (w/r) is equal to the marginal rate of technical substitution of labor for capital (dK/dL). That is, the slope of factor cost line is set equal to that of the isoquant. Algebraically, this condition of the optimal factor proportion is expressed as:

$$- (dK/dL) = (w/r)$$

For clothing industry, this expression can be written in terms of input-output coefficients as follows due to the assumption of constant return to scale.

$$- \frac{da_{KC}}{da_{LC}} = \frac{w}{r}$$

Rearrange and manipulate algebraically this equation to obtain[14]:

$$v_{LC}\hat{a}_{LC} + v_{KC}\hat{a}_{KC} = 0 \qquad (8.1)$$

In a similar fashion, for the food indusry,

$$v_{LF}\hat{a}_{LF} + v_{KF}\hat{a}_{KF} = 0 \qquad (8.2)$$

These two expressions, (8.1) and (8.2), make the competitive profit equations of change with variable input-output coefficients identical to those equations in the fixed coefficient case:

$$v_{LC}\hat{w} + v_{KC}\hat{r} = \hat{p}_C \qquad (3.3)$$
$$v_{LF}\hat{w} + v_{KF}\hat{r} = \hat{p}_F \qquad (4.3)$$

Therefore, the same conclusion for the magnification effect of commodity prices on factor rewards can be made as in the case of fixed coefficients.

If $\hat{p}_C > \hat{p}_F$,

then $\hat{w} > \hat{p}_C > \hat{p}_F > \hat{r}$

and vice versa. The validity of the Stolper-Samuelson Theorem is proved in the more general case of production technology with variable input-output coefficients too.

Next, turn to the relationship between the relative

change in factor endowments and commodity outputs that is expressed by the equations, (1.1) and (2.1). These equations will not reduce to a simple form like the competitive-profit equations of change because the bracketed expressions, $(u_{LC}\hat{a}_{LC} + u_{LF}\hat{a}_{LF})$ and $(u_{KC}\hat{a}_{KC} + u_{KF}\hat{a}_{KF})$ are not zero. Let us look at the expression $(u_{LC}\hat{a}_{LC} + u_{LF}\hat{a}_{LF})$. Suppose wage rates rise relatively higher than rental rates. Both industries respond to this relative change in factor prices by changing factor proportion employed in production, using less labor and more capital than previously. Hence, the labor input-output coefficients become smaller. In other words, both $\hat{a}_{LC}$ and $\hat{a}_{LF}$ are negative. Then, since the u's are positive fractions, the bracketed term $(u_{LC}\hat{a}_{LC} + u_{LF}\hat{a}_{LF})$ is necessarily less than zero. In the opposite case, where the wage/rental ratio declines, the bracketed term becomes positive. In both cases, the bracketed term cannot be zero. The same reasoning applies equally to the equation (4.1) for food industry.

As discussed earlier, with variable coefficients, substitution between the two factors is technically feasible in response to a change in factor price ratio. The extent of the change in factor ratio resulting from the change in relative factor prices depends on the elasticity of factor substitution. Suppose $e_C$ and $e_F$ stand for the elasticities of substitution of capital for labor in clothing and food

industry respectively. More specifically, $e_C$ is defined as a percentage change in the capital/labor ratio in clothing industry that is associated with a percentage change in the wage/rent ratio. In the relative terms, this definition is expressed as:

$$e_C = \frac{\hat{a}_{KC} - \hat{a}_{LC}}{\hat{w} - \hat{r}} \qquad (9)$$

$$e_F = \frac{\hat{a}_{KF} - \hat{a}_{LF}}{\hat{w} - \hat{r}} \qquad (10)$$

As mentioned earlier, if wage rates rise relatively higher than rental rate ($\hat{w} > \hat{r}$), both industries use less labor and more capital, that is, $\hat{a}_{KC} > \hat{a}_{LC}$ and $\hat{a}_{KF} > \hat{a}_{LF}$. In the opposite case, where the wage/rent ratio declines ($\hat{w} < \hat{r}$), the labor/capital ratio in producing each commodity rises ($\hat{a}_{KC} < \hat{a}_{LC}$ and $\hat{a}_{KF} < \hat{a}_{LF}$). That is, both numerator and denominatior in each expression of $e_C$ and $e_F$ have the same sign whether the wage/rent ratio rises or declines in both industries. Hence, $e_C$ and $e_F$ must be positive.

Combining the expression (9) with the equation (8.1), we can get the explicit solutions for $\hat{a}_{LC}$ and $\hat{a}_{KC}$ as functions of the relative factor price changes, $(\hat{w} - \hat{r})$, as follows.(15)

$$\begin{aligned} \hat{a}_{LC} &= - v_{KC} e_C (\hat{w} - \hat{r}) \\ \hat{a}_{KC} &= v_{LC} e_C (\hat{w} - \hat{r}) \end{aligned} \qquad (11)$$

The solutions for $\hat{a}_{LF}$ and $\hat{a}_{KF}$ are obtained in a similar fashion as follows.

$$\begin{aligned} \hat{a}_{LF} &= - v_{KF}e_F(\hat{w} - \hat{r}) \\ \hat{a}_{KF} &= v_{LF}e_F(\hat{w} - \hat{r}) \end{aligned} \qquad (12)$$

If we substitute the solutions for the $\hat{a}$'s in (11) and (12) into the equations, (3.3) and (4.3) respectively, then the two full-employment equations of changes are expressed as[16]:

$$\begin{aligned} u_{LC}\hat{Y}_C + u_{LF}\hat{Y}_F &= \hat{L} + (\hat{w} - \hat{r})(u_{LC}v_{KC}e_C + u_{LF}v_{KF}e_F) \\ u_{KC}\hat{Y}_C + u_{KF}\hat{Y}_F &= \hat{K} - (\hat{w} - \hat{r})(u_{KC}v_{LC}e_C + u_{KF}v_{LF}e_F) \end{aligned}$$

Simplify the equations with appropriate notation to obtain:

$$u_{LC}\hat{Y}_C + u_{LF}\hat{Y}_F = \hat{L} + q_L(\hat{w} - \hat{r}) \qquad (1.3)$$
$$u_{KC}\hat{Y}_C + u_{KF}\hat{Y}_F = \hat{K} - q_K(\hat{w} - \hat{r}) \qquad (2.3)$$

where

$$q_L = (u_{LC}v_{KC}e_C + u_{LF}v_{KF}e_F)$$
$$q_K = (u_{KC}v_{LC}e_C + u_{KF}v_{LF}e_F)$$

Both $q_L$ and $q_K$ have positive values because all the terms, the u's, v's, and the e's are positive. The positive value of the $q_L$ implies, together with the equation (1.3), that the rise in the wage/rent ratio has the same effect on outputs as an increase in the labor endowment has. If wage rates rise more than rental rates, both industry would substitute capital for labor, sloughing off labor. As a

result, there is an increase in available labor like an increase in the labor endowment. Then the output of clothing industry, which is labor intensive, must expand in order to maintain full employment. This reasoning holds equally in the case where the wage/rent ratio declines.

In Section 2, the factor price ratio was expressed as a function of the commodity price ratio as follows.

$$(\hat{w} - \hat{r}) = \frac{1}{|V|} (\hat{p}_C - \hat{p}_F) \qquad (5.2)$$

If the expression (5.2) is substituted into the equations, (1.3) and (2.3), then

$$u_{LC}\hat{Y}_C + u_{LF}\hat{Y}_F = \hat{L} + \frac{q_L}{|V|}(\hat{p}_C - \hat{p}_F) \qquad (1.4)$$

$$u_{KC}\hat{Y}_C + u_{KF}\hat{Y}_F = \hat{K} - \frac{q_K}{|V|}(\hat{p}_C - \hat{p}_F) \qquad (2.4)$$

Also, $|V|$ was shown to be a positive fraction under the assumption of factor intensity that the clothing industry is labor intensive. Hence, these two equations state that a change in the relative commodity prices has the same effect on commodity output as the change in factor endowment does.

Suppose that commodity prices are held constant, that is, $\hat{p}_C = \hat{p}_F = 0$. Then the equations, (1.4) and (2.4), reduce to the following simple form:

$$u_{LC}\hat{Y}_C + u_{LF}\hat{Y}_F = \hat{L} \qquad (1.5)$$

$$u_{KC}\hat{Y}_C + u_{KF}\hat{Y}_F = \hat{K} \qquad (2.5)$$

Therefore, if commodity prices are held constant, the relationship in the relative changes between factor endowments and commodity outputs is the same as in the case of fixed coefficients. In other words, the relative changes in factor endowments is equal to a positive weighted average of relative output changes. Likewise, if:

$$\hat{L} > \hat{K}$$

then:

$$\hat{Y}_C > \hat{L} > \hat{K} > \hat{Y}_K$$

Hence, with the constant commodity prices, the magnification effect of factor endowments upon commodity outputs and, thus, the Rybczynski Theorem remain valid.

## 6. Conclusion

The relationships between factor endowments and commodity outputs on the one hand and between commodity and factor prices on the other are two basic relationships in the pure theory of international trade(Jones, Dec. 1965). Any change in factor endowments leads to a corresponding change in commodity outputs, and a change in commdity prices is responded by achange in factor rewards. The proposition of the magnification effect, which was first introduced by Professor Jones in 1965, emphasizes and details the quantitative aspects of the two basic relationships. Using a

simple general equilibrium model composed of two factors and two commodities, the proposition shows how the change in the factor endowment ratio results in the more magnified change in the commodity output ratio on the one hand, and how the relative change in commodity prices leads to the more magnified change in factor rewards on the other.

The proposition of the magnification effect also clarifies and provides a firm theoretical ground for the two core theorems of the Heckscher-Ohlin model. The magnification effect of factor endowments on commodity outputs shows a broader causal relationship existing the two variables than the Rybczyski Theorem states, supplying the theorem with a solid proof. The magnification effect of commodity prices on factor rewards enhances the validity of the Stolper-Samuelson Theorem, explaining how a change in commodity prices leads to a corresponding change in factor returns regardless of the consumption pattern of factor owners. In this way, the proposition of the magnification effect takes on an essential part in the body of the theory of international trade, proving the two core theorems of the Heckscher-Ohlin Model as well as explaining the quantitative aspect of the two basic relationships of the theory.

## NOTES

1. The debate proceeded between the two conflicting view concerning the effect of trade on the real income of laborers, that is, between the popular notion that the American workers must be protected against the competition of cheap foreign labor and the scholastic view that real income of workers are not affected by trade but by labor productivity. For more detailed information, refer to the original paper of Stolper and Samuelson.

2. This expression is drawn from Kindleberger and Lindert(1978).

3. Jones and Neary termed the four theorem - the Heckscher-Ohlin, the Factor Price Equalization, the Stolper-Samuelson, and the Rybczynski Theorem - as the core propositions of the Heckscher-Ohlin Model.

4. According to Professor Jones, the technolgy is described by the columns of the following coefficient matrix.

$$A = \begin{pmatrix} a_{LC} & a_{LF} \\ a_{KC} & a_{KF} \end{pmatrix}$$

5. Without full employment conditions, the economy's aggregate demand for each factor of production is not equal

to the total factor supply in a competitive equilibrium. Thus, the equilibrium conditions are presented in this case as inequalities to allow for the existence of resources in excess supply as follows.

$$a_{LC}y_C + a_{LF}y_F \leq L$$
$$a_{KC}y_C + a_{KF}y_F \leq K$$

6. The equilibrium conditions of the economy under perfect competition are usually described as the inequalities between unit cost and price in each commodity as follows.

$$a_{LC}w + a_{KC}r \geq p_C$$
$$a_{LF}w + a_{KF}r \geq p_F$$

Price can never exceed unit cost in a competitive equilibrium. However, unit cost may exceed price if all producers in a particular industry leave the industry, that is, if there is a complete specialization in the economy where one of the two industries is shut down. Since we assume the economy with incomplete specialization where both industries operate, the equilibrium conditions are described as equality between unit cost and price.

7. $a_{LC}dy_C + a_{LF}dy_F = dL - (y_C da_{LC} + y_F da_{LF})$

Divide both sides of the equation by L:

$$\frac{a_{LC}}{L}dy_C + \frac{a_{LF}}{L}dy_F = \frac{dL}{L} - (\frac{y_C}{L}da_{LC} + \frac{y_F}{L}da_{LF})$$

Manipulate this expression to obtain:

$$\frac{a_{LC}y_C}{L}.\frac{dy_C}{y_C} + \frac{a_{LF}y_F}{L}.\frac{dy_F}{y_F} = \frac{dL}{L} - (\frac{a_{LC}y_C}{L}.\frac{da_{LC}}{a_{LC}} + \frac{a_{LF}y_F}{L}.\frac{da_{LF}}{a_{LF}})$$

8. I used the same notation throughout this paper as Professor Jones did in his publications, Jones(Aug. 1965), Jones(Dec. 1965), Caves and Jones(1973), and Jones and Neary(1984).

9.
$$\frac{a_{LC}y_C}{L} + \frac{a_{LF}y_F}{L} = \frac{a_{LC}y_C + a_{LF}y_F}{L}$$

By the equation (1):

$$\frac{a_{LC}y_C}{L} + \frac{a_{LF}y_F}{L} = \frac{L}{L}$$

10.
$$\frac{a_{LC}w}{p_C} + \frac{a_{KC}r}{p_C} = \frac{a_{LC}w + a_{KC}r}{p_C}$$

By the equation (3):

$$\frac{a_{LC}w}{p_C} + \frac{a_{KC}r}{p_C} = \frac{p_C}{p_C}$$

Hence, the two terms add to unity.

11. The coefficient matrix of the equations, (1.2) and

(2.2), is expressed as:

$$U = \begin{pmatrix} u_{LC} & u_{LF} \\ u_{KC} & u_{KF} \end{pmatrix}$$

The determinant of this matrix is:

$$|U| = u_{LC} \cdot u_{KF} - u_{KC} \cdot u_{LF}$$

Since $u_{LF} = (1 - u_{LC})$ and $u_{KF} = (1 - u_{KC})$,:

$$|U| = u_{LC}(1 - u_{KC}) - u_{KC}(1 - u_{LC})$$

Rearrange the terms of the right side of equation to obtain:

$$|U| = (u_{LC} - u_{KC})$$

12. If the clothing industry is labor intensive, then

$$\frac{L_C}{K_C} > \frac{L_F}{K_F}$$

where the $L_i$'s and the $K_i$'s denote the production factors used to produce the two commodities respectively. Under the assumption of homogeneous production function of degree one, this inequality is expressed as:

$$\frac{a_{LC} \cdot Y_C}{a_{KC} \cdot Y_C} > \frac{a_{LF} \cdot Y_F}{a_{KF} \cdot Y_F}$$

Hence,:

$$\frac{a_{LC}}{a_{KC}} > \frac{a_{LF}}{a_{KF}}$$

13. The coefficient matrix of the equations, (3.2) and (4.2) is expressed as:

$$V = \begin{bmatrix} v_{LC} & v_{KC} \\ v_{LF} & v_{KF} \end{bmatrix}$$

The determinant of this matrix is:

$$|V| = v_{LC} \cdot v_{KF} - v_{KC} \cdot v_{LF}$$

Apply $v_{KF} = (1 - v_{LF})$ and $v_{KC} = (1 - v_{LC})$ to this expression:

$$|V| = v_{LC}(1 - v_{LF}) - v_{LF}(1 - v_{LC})$$

Rearrange the terms of the right side of equation:

$$|V| = (v_{LC} - v_{LF})$$

14. The expression for the optimal factor proportion is put in relative terms through the following process. The conditions for the optimal factor proportion is expressed as:

$$-\frac{da_{KC}}{da_{LC}} = \frac{w}{r}$$

Multiply both sides of the equaion by $da_{LC}$ and r to

obtain:

$$- r^{\cdot}da_{KC} = w^{\cdot}da_{LC}$$

Add $r^{\cdot}da_{KC}$ to the both sides. Then:

$$w^{\cdot}da_{LC} + r^{\cdot}da_{KC} = 0$$

Divide the both sides by $p_C$:

$$\frac{w}{p_C}da_{LC} + \frac{r}{p_C}da_{KC} = 0$$

By a simple algebraic manipulation:

$$\frac{w^{\cdot}a_{LC}}{p_C}.\frac{da_{LC}}{a_{LC}} + \frac{r^{\cdot}a_{KC}}{p_C}.\frac{da_{KC}}{a_{KC}} = 0$$

Since $v_{LC}$ and $v_{KC}$ denote $\frac{w^{\cdot}a_{LC}}{p_C}$ and $\frac{r^{\cdot}a_{KC}}{p_C}$ respectively,:

$$v_{LC}\hat{a}_{LC} + v_{KC}\hat{a}_{KC} = 0$$

15. The solution for $\hat{a}_{LC}$ can be obtained by the following procedure. Reproduce the equation (8.1):

$$v_{LC}\hat{a}_{LC} + v_{KC}\hat{a}_{KC} = 0$$

Subtract $v_{LC}\hat{a}_{LC}$ from both sides:

$$v_{LC}\hat{a}_{LC} = - v_{KC}\hat{a}_{KC}$$

Divide both sides by $v_{KC}$:

$$\hat{a}_{KC} = - \frac{v_{LC}}{v_{KC}}\hat{a}_{LC}$$

The definition of $e_C$ is expressed as:

$$e_C = \frac{\hat{a}_{KC} - \hat{a}_{LC}}{\hat{w} - \hat{r}}$$

Multiply both sides by $-(\hat{w} - \hat{r})$:

$$\hat{a}_{KC} - \hat{a}_{LC} = - e_C(\hat{w} - \hat{r})$$

Substitute $- \frac{v_{LC}}{v_{KC}}\hat{a}_{LC}$ for $\hat{a}_{KC}$:

$$\hat{a}_{LC} + \frac{v_{LC}}{v_{KC}}\hat{a}_{LC} = - e_C(\hat{w} - \hat{r})$$

By manipulation of the left side of the equation:

$$\frac{v_{KC} + v_{LC}}{v_{KC}}\hat{a}_{LC} = - e_C(\hat{w} - \hat{r})$$

Since $v_{LC} + v_{KC} = 1$,:

$$\frac{1}{v_{KC}}\hat{a}_{LC} = - e_C(\hat{w} - \hat{r})$$

Multiply both sides by $v_{KC}$:

$$\hat{a}_{LC} = - v_{KC} \cdot e_C(\hat{w} - \hat{r})$$

The solution for $\hat{a}_{KC}$ can be obtained similarly.

16. The bracketed expression in the equation (3.3) is:

$$u_{LC}\hat{a}_{LC} + u_{LF}\hat{a}_{LF}$$

Substitute the solutions for the $\hat{a}$'s in (11) and (12) in this expression:

$$u_{LC}\{-v_{KC}e_C(\hat{w}-\hat{r})\} + u_{LF}\{-v_{KF}e_F(\hat{w}-\hat{r})\}$$

Rearrange this expression into:

$$-(\hat{w}-\hat{r})(u_{LC}v_{KC}e_C + u_{LF}v_{KF}e_F)$$

The bracketed expression is the equation (1.2) can be expressed in a similar way as:

$$(\hat{w}-\hat{r})(u_{KC}v_{KC}e_C + u_{KF}v_{LF}e_F)$$

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# ALGEBRAIC ANALYSIS OF THE MAGNIFICATION EFFECT IN THE PURE THEORY OF INTERNATIONAL TRADE

by

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B.A., Seoul National University, Korea, 1980

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF ARTS

Department of Economics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1987

The relationships between factor endowments and commodity outputs on the one hand and between commodity and factor prices on the other are two basic relationships in the pure theory of international trade. In the model of simple general equilibrium, which is commonly used in the theoretical analysis of international trade, factor endowments are linked to commodity outputs under the assumption of full employment, and commodity prices are related to factor prices under the assumption of perfect competition. The theory of international trade describes the former relationship for the physical quantities that a change in the ratio of factor endowments in a country causes a change in the composition of the commodity outputs produced in that country. In a similar fashion, the latter relationship for the prices of commodities and factors is described that a change in price ratio of commodities results in a change in the relative returns to productive factors. This report deals with the question of how large the impact of the changes in factor endowments is on the composition of commodity outputs on the one hand and that of the changes in commodity prices is on the factor rewards.

Using the simple production model composed of two factors and two commodities, and assuming that production technology is characterized by constant return to scale, this report shows that there are magnification effects in the two basic relationships in the theory of international trade. A

relative change in factor endowments results in a more magnified change in the composition of commodity outputs that causes it. Similarly, the resulting change in the ratio of factor rewards is more magnified than the causing change in the price ratio of commodities.

In addition, it is shown that the proposition of the magnification effect provides a firm theoretical ground for the two core theorems of the Heckscher-Ohlin Model - the Rybczynski Theorem and the Stolper-Samuelson Theorem. The magnification effect of factor endowment on commodity output evidences that an increase in one productive factor with constant endowment of the other results in a greater than proportionate increase in the total output of the commodity embodying the increased factor intensively and in an actual decline in the total output of the other good as the Rybczynski Theorem states. Similarly, the magnification effect of commodity prices on factor rewards shows clearly that a change in the price ratio of commodities leads to a more magnified change in the price ratio of factors as stated by the Stolper-Samuelson Theorem.